

Government College of Engineering and Research, Avasari(Khurd)

Department: Mechanical Engineering

Learning Resource Material (LRM)

Name of the course: Mechanical System Design **Course Code:** 402048

Name of the faculty: J. M. Arackal **Class:** BE(Mech)

SYLLABUS(Unit 4)

Unit 4: Design of Cylinders and Pressure Vessels

Design of Cylinders: Thin and thick cylinders, Lamé's equation, Clavarino's and Bernie's equations, design of hydraulic and pneumatic cylinders, auto-frettage and compound cylinders,(No Derivation) gasketed joints in cylindrical vessels (No derivation).

Design of Pressure vessel : Modes of failures in pressure vessels, unfired pressure vessels, classification of pressure vessels as per I. 2825 - categories and types of welded joints, weld joint efficiency, stresses induced in pressure vessels, materials for pressure vessel, thickness of cylindrical shells and design of end closures as per code, nozzles and openings in pressure vessels, reinforcement of openings in shell and end closures - area compensation method, types of vessel supports (theoretical treatment only).

Lecture Plan format:**Name of the course:** Mechanical System Design **Course Code** 402048

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Unit No	Lecture No.	Topics to be covered	Text/Reference Book/ Web Reference
		UNIT 4	
4	1	Thin and thick cylinders	1
4	2	Lame's equation, Clavarino and Bernie's equations	1
4	3	Design of hydraulic and pneumatic cylinders	1
4	4	Auto-frettage and compound cylinders	1
4	5	Gasketed joints in cylindrical vessels	1
4	6	Modes of failures in pressure vessels, unfired pressure vessels, classification of pressure vessels as per I. S. 2825 - categories and types of welded joints	1
4	7	thickness of cylindrical shells and design of end closures as per code	1
4	8	nozzles and openings in pressure vessels	1

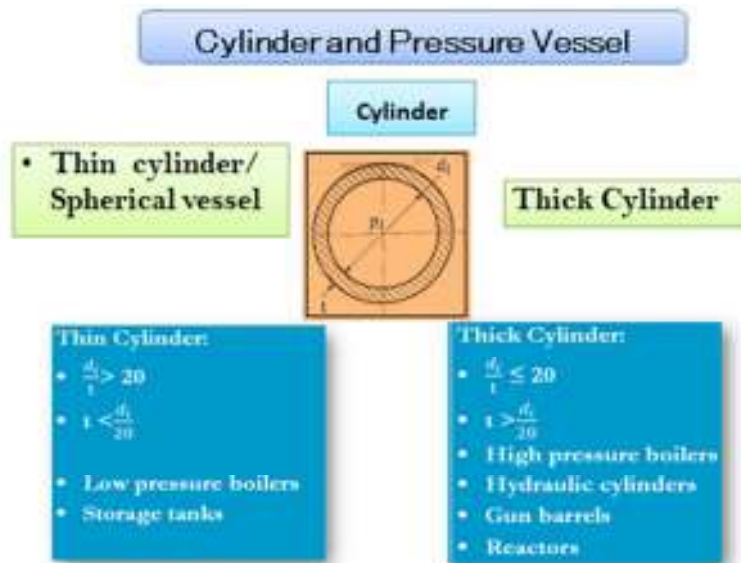
List of Text Books /Reference Books/ Web Reference

1-Bhandari V.B. —*Design of Machine Elements*||, Tata McGraw Hill Pub. Co. Ltd.

2-R.K. Jain- *Machine Design*, Khanna Publishers

3-Johnson R.C., —*Mechanical Design Synthesis with Optimization Applications*||, Von Nostrand Reynold Pub

UNIT 4- DESIGN OF CYLINDERS AND PRESSURE VESSEL



Stresses in Thin Cylinder

1] Circumferential /Hoop stress/Tangential:
Exerted *circumferentially* on every particle in the cylinder wall.
 - Can be imagined as a band surrounding a barrel.
 • When barrel expands, the band stretches and undergoes stress.



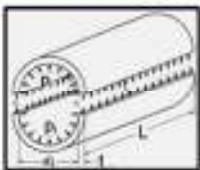
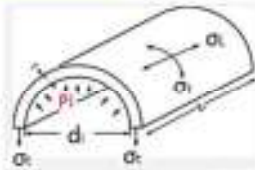


2] Longitudinal stress:
Parallel to the axis of cylindrical



3] Radial stress: (Compressive)
Caused by the design pressures acting through the wall thickness (neglected). As P small

Tangential/Circumferential/Hoop stress

- $(p_i)(\text{Projected area}) = (\sigma_c)(\text{Resisting Area})$
- $(p_i)(d_i \times L) = (\sigma_c)(2t \times L)$

Balance equation
(Induced stress = Resisting stress)

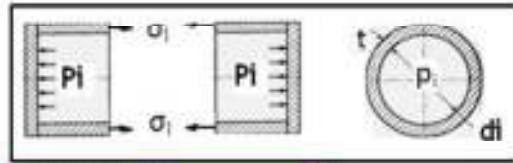
• $\sigma_c = \frac{p_i d_i}{2t}$

• $\sigma_c = \frac{p_i d_i}{2t \eta_L}$

Assumption:
Hoop stress is uniformly distributed through t (because t is small)

$\eta_L = \text{Longitudinal joint efficiency}$

Longitudinal/Axial Stress



Balance equation
Induced stress = Resisting stress

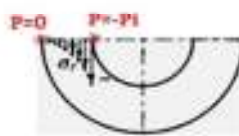
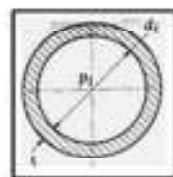
$$p_i (\text{Projected area}) = \sigma_l (\text{Resisting Area})$$

$$p_i \left(\frac{\pi}{4} d_i^2 \right) = \sigma_l (\pi d_i t)$$

$$\sigma_l = \frac{p_i d_i}{4t} \quad \sigma_l = \frac{p_i d_i}{4t \eta_c}$$

η_c = Circumferential joint efficiency

Radial Stress in Thin Cylinder (σ_r)



$\sigma_r = p_i$ (Inner radius)
 $= 0$ (Outer radius)

p_i : Very Less : σ_r is neglected

t : Less

Assumption:
Radial stress is neglected as P is very small

Principal Stresses in Thin Cylinder

$$\sigma_t = \frac{p_i d_i}{2t \eta_l}$$

$$\sigma_l = \frac{p_i d_i}{4t \eta_c}$$

σ_t
 σ_l
 σ_r

$\sigma_t > \sigma_l$ SO: thickness is calculated using tangential stress

$$t = \frac{p_i d_i}{2\sigma_t \eta_l}$$

$\sigma_t = \sigma_{all}$ (Based on failure theory)

Stress in Spherical vessel



- $(p_i)(\text{Projected area}) = (\sigma)(\text{Resisting Area})$
- $(p_i)(\frac{\pi}{4}d_i^2) = (\sigma)(\pi d_i t)$

• $\sigma = \frac{p_i d_i}{4t}$ • $\sigma = \frac{p_i d_i}{4t \eta_{jc}}$
 $\eta_{jc} = \text{joint efficiency}$

Spherical pressure vessel has **twice** the strength of a Cylindrical pressure vessel

For Spherical vessel $\sigma = \sigma_t = \sigma_r$

Seamless cylinder.
 Storage capacity = 0.25 m³, $P_i = 20 \text{ Mpa}$, $L = 2d_i$,
 20C8 ($S_{ut} = 390 \text{ Mpa}$), $FoS = 2.5$. Dimensions?

Q

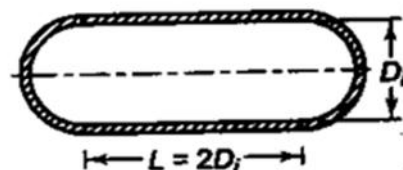
$$V = \frac{\pi}{4} d_i^2 L = 0.25 \text{ m}^3 \quad \Rightarrow \quad L = 2 d_i \quad \Rightarrow \quad d_i = 0.25 \text{ m} = 250 \text{ mm}$$

$$t = \frac{P_i d_i}{\sigma t} \quad \Rightarrow \quad t = 16 \text{ mm} \quad \Rightarrow \quad L = 320 \text{ mm}$$

$$\sigma t = \frac{S_{ut}}{FoS} = \frac{390}{2.5} = 156$$

Q

Air receiver:
 Storage capacity: 0.25 m³
 Operating pressure: $P_i = 5 \text{ Mpa}$
 10C8 ($S_{ut} = 340 \text{ Mpa}$)
 $FoS = 4$
 Neglect weld efficiency.
 Dimensions of receiver:?



$$V = \frac{\pi}{4} d_i^2 L + \frac{\pi}{6} d_i^3 \quad L = 2 d_i$$

$$d_i = 0.492 \text{ m} = 500 \text{ mm}$$

$$L = 1000 \text{ mm}$$

$$\sigma t = \frac{S_{ut}}{FoS} = \frac{340}{4} = 85$$

Cylinder

$$\sigma t = \frac{p_i d_i}{2 t} \quad t = 17.7 = 15 \text{ mm}$$

Sprical head

$$\sigma t = \frac{p_i d_i}{4 t} \quad t = 7.35 = 8 \text{ mm}$$

Stresses in Thick Cylinder

CBS

- $\frac{d_i}{t} \leq 20 \quad t > \frac{d_i}{20}$
- High pressure boilers
- Hydraulic cylinders
- Gun barrels
- Reactors

Thin Cylinder Assumption:

- Circumferential Stress is uniformly distributed over the thickness
- Radial Stress is neglected: Since for thin cylinder p_i is small

Cylinder with Internal Pressure (P_i)

$$\sigma_r = - \frac{p_i d_i^2}{[d_o^2 - d_i^2]} \left[\frac{d_o^2}{4r^2} - 1 \right] \quad \text{Compressive stress}$$

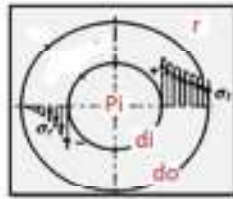
$$\sigma_t = \frac{p_i d_i^2}{[d_o^2 - d_i^2]} \left[\frac{d_o^2}{4r^2} + 1 \right]$$



$$\sigma_l = \frac{p_i (d_i^2)}{[d_o^2 - d_i^2]}$$

$$\sigma_l = \frac{p_i d_i}{4t}$$

Principal stresses



$r = \frac{di}{2}$	$r = \frac{do}{2}$
$\sigma_r = -Pi$	$\sigma_r = 0$
$\sigma_t = \frac{Pi (do^2 + di^2)}{[do^2 - di^2]}$	$\sigma_t = \frac{2Pi (di^2)}{[do^2 - di^2]}$

$$\sigma_t > \sigma_l \gg \gg \sigma_r$$

Max Principal stresses

$$\sigma_t = \frac{Pi (do^2 + di^2)}{[do^2 - di^2]} = \sigma_{max}$$

Min Principal stresses

$$\sigma_r = -Pi = \sigma_{min}$$

Lame's Equation

Wall thickness of shell $t =$
Theories of failure

- Brittle material (e.g. CI)
- Based on Maximum principal stress theory of failure
- Maximum principal stress = Allowable (permissible) stress of material
- $\sigma_{max} = \sigma_{all}$

A) Maximum principal stress (σ_{max})

$$\sigma_{max} = \sigma_t = \frac{Pi (do^2 + di^2)}{[do^2 - di^2]}$$

$$\sigma_{max} = \sigma_t = \frac{Pi (do^2 + di^2)}{[do^2 - di^2]} = \sigma_{all} = \frac{\sigma_{ult}}{FoS}$$

Substitute $t = \frac{do - di}{2}$

$$t = \frac{di}{2} \left(\sqrt{\frac{\sigma_{all} + Pi}{\sigma_{all} - Pi}} - 1 \right)$$

Clavarino's & Birnie's equation / St Venants theory

- Ductile material (e.g. Steel)
- Based on Maximum Strain energy theory of failure
- Maximum Strain = Yield point strain of material (allowable)
- $\epsilon_{max} = \epsilon_{all}$

Clavarino's Eq
Closed cylinder

$$\sigma_t = \frac{Pi (do^2 + di^2)}{[do^2 - di^2]}$$

$$\sigma_l = \frac{Pi (di^2)}{[do^2 - di^2]}$$

$$\sigma_r = -Pi$$

$$t = \frac{do - di}{2}$$

As $\sigma_t \gg \sigma_l$ & σ_r

$$\epsilon_{max} = \epsilon_t = \frac{\sigma_t}{E} - \mu \frac{\sigma_r}{E} - \mu \frac{\sigma_l}{E}$$

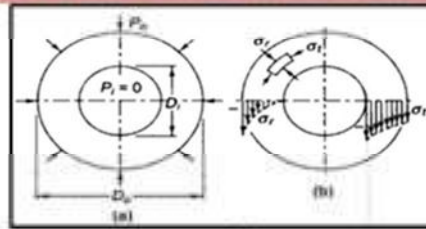
$$\epsilon_{all} = \frac{\sigma_{all}}{E}$$

$$\frac{\sigma_t}{E} - \mu \frac{\sigma_r}{E} - \mu \frac{\sigma_l}{E} = \frac{\sigma_{all}}{E}$$

$$\sigma_{all} = \frac{\sigma_{yt}}{FoS}$$

$$t = \frac{di}{2} \left(\sqrt{\frac{\sigma_{all} + (1 - 2\mu)Pi}{\sigma_{all} - (1 - \mu)Pi}} - 1 \right)$$

Cylinder with External Pressure (Po)



$$\sigma_r = -\frac{P_o d_o^2}{[d_o^2 - d_i^2]} \left[1 - \frac{d_i^2}{4r^2} \right]$$

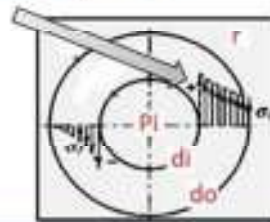
$$\sigma_t = -\frac{P_o d_o^2}{[d_o^2 - d_i^2]} \left[1 + \frac{d_i^2}{4r^2} \right]$$

$r = \frac{d_i}{2}$	$r = \frac{d_o}{2}$
$\sigma_r = 0$	$\sigma_r = -P_o$
$\sigma_t = -\frac{2 P_o [d_o^2]}{[d_o^2 - d_i^2]}$	$\sigma_t = -\frac{P_o [d_o^2 + d_i^2]}{[d_o^2 - d_i^2]}$

Autofrettage Pre-Stressing

When subjected to P_i ,

Hoop stress σ_t limits pressure capacity



Autofrettage is method to increase the pressure capacity of cylinder

Used for HP Cylinder, Gun Barrels

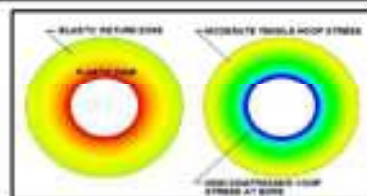
1] Overloading method of Autofrettage

- Cylinder subjected to immense pressure, which causes the internal parts of the vessel to yield, thus resulting in internal compressive residual stresses.

It increases pressure capacity of cylinder

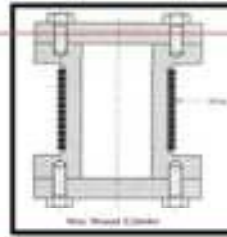
Residual compressive stresses close the cracks

For same thickness cylinder can be used for P_i more than designed.



2) Wire wound method of Autofrettage

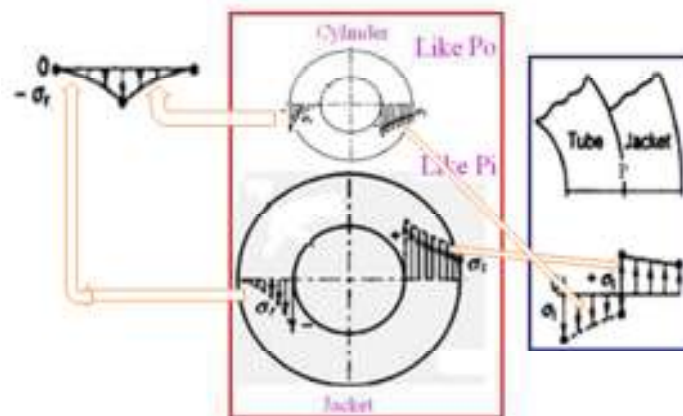
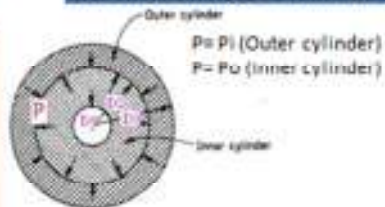
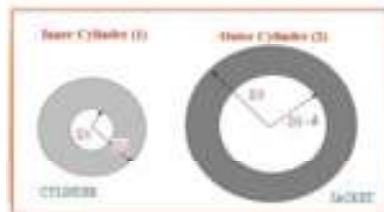
Wire under tension is closely wound the cylinder results in residual compressive stresses



3) Compounding of cylinders

Two concentric cylinder with outer cylinder shrunk onto inner one. It induces residual compressive stresses on inner cylinder

3) Compounding of cylinders



Deformations in jacket and cylinder



$$\delta_i = \frac{D_2 P}{E} \left[\frac{(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} + \mu \right] \quad \delta_e = -\frac{D_2 P}{E} \left[\frac{(D_3^2 + D_1^2)}{(D_3^2 - D_1^2)} - \mu \right]$$

$$\delta = \frac{PD_2}{E} \left[\frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_3^2 - D_1^2)} \right]$$

The shrinkage pressure P

A high-pressure cylinder consists of a steel tube with inner and outer diameters of 20 and 40 mm respectively. It is jacketed by an outer steel tube, having an outer diameter of 60 mm. The tubes are assembled by a shrinking process in such a way that maximum principal stress induced in any tube is limited to 100 N/mm². Calculate the shrinkage pressure and original dimensions of the tubes ($E = 207 \text{ kN/mm}^2$).

$$D_1 = 20 \text{ mm} \quad D_2 = 40 \text{ mm} \quad D_3 = 60 \text{ mm}$$

$$\sigma_{\max} = 100 \text{ N/mm}^2 \quad E = 207 \text{ kN/mm}^2$$

Shrinkage pressure

$$\sigma_t = \frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} \quad \text{or} \quad 100 = \frac{P(60^2 + 40^2)}{(60^2 - 40^2)}$$

$$P = 38.46 \text{ N/mm}^2$$

Stress due to Shrink Pressure [P=38.46]

Jacket

$$\sigma_r = -\frac{PD_2^2}{(D_3^2 - D_2^2)} \left[\frac{D_3^2}{4r^2} - 1 \right] = -30.77 \left[\left(\frac{30}{r} \right)^2 - 1 \right]$$

$$\sigma_t = +\frac{PD_2^2}{(D_3^2 - D_2^2)} \left[\frac{D_3^2}{4r^2} + 1 \right] = +30.77 \left[\left(\frac{30}{r} \right)^2 + 1 \right]$$

R	10	15	20
σ_R	0	-28	-38
σ_t	103	74	64

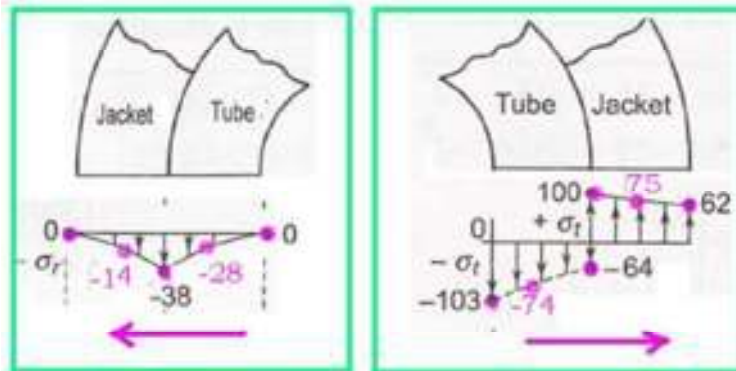
Stress due to Shrink Pressure [P=38.46]

Cylinder

$$\sigma_r = -\frac{PD_2^2}{(D_3^2 - D_1^2)} \left[1 - \frac{D_3^2}{4r^2} \right] = -51.28 \left[1 - \left(\frac{10}{r} \right)^2 \right]$$

$$\sigma_t = +\frac{PD_2^2}{(D_3^2 - D_1^2)} \left[1 + \frac{D_3^2}{4r^2} \right] = +51.28 \left[1 + \left(\frac{10}{r} \right)^2 \right]$$

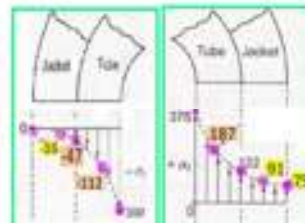
R	20	25	30
σ_R	-38	-14	-0
σ_t	100	75	62



Stress due to Internal Pressure [Pi=300]

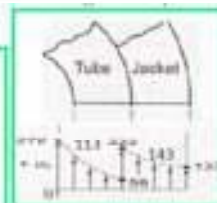
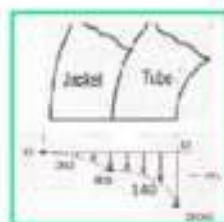
$$\sigma_r = -\frac{P_i D_i^2}{(D_o^2 - D_i^2)} \left[\frac{D_o^2}{4r^2} - 1 \right] = -37.5 \left[\left(\frac{30}{r} \right)^2 - 1 \right]$$

$$\sigma_t = +\frac{P_i D_i^2}{(D_o^2 - D_i^2)} \left[\frac{D_o^2}{4r^2} + 1 \right] = +37.5 \left[\left(\frac{30}{r} \right)^2 + 1 \right]$$



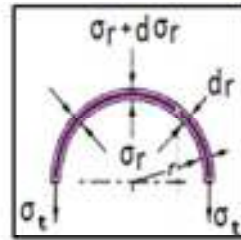
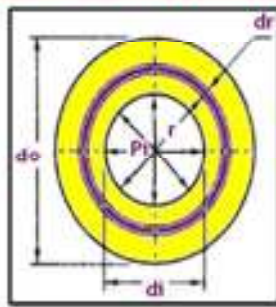
	Cylinder			Jacket		
R	10	15	20	20	25	30
σ _r	-300	-112	-47	-47	-16	0
σ _t	375	187	122	122	91	75

		Cylinder			Jacket		
GR	Pi	-300	-112	-47	-47	-16	0
	P	0	28	28	38	14	0
	R	-300	-140	-85	-85	-30	0
GT	Pi	375	187	122	122	91	75
	P	-103	-74	-64	100	75	62
	R	272	113	58	222	143	137



Cylinder with Internal Pressure (Pi)

CBS



$$[\sigma_t 2dr l] + [2(r + dr)l(\sigma_r + d\sigma_r)] = [2rl \sigma_r]$$

Neglecting $(dr \times d\sigma_r)$

$$(\sigma_t + \sigma_r) + r \frac{d}{dr}(\sigma_r) = 0 \quad \text{eq. 1}$$

ϵ_l is constant over the thickness

CBS

$$\epsilon_l = \frac{\sigma_l}{E} + \mu \frac{\sigma_r}{E} - \mu \frac{\sigma_t}{E} \quad (\sigma_r - \sigma_t) = \frac{E}{\mu} \left(\epsilon_l - \frac{\sigma_l}{E} \right)$$

$$(\sigma_r - \sigma_t) = -2A \quad \text{eq. 2}$$

From eqn 1 and 2

$$2\sigma_r + r \frac{d}{dr}(\sigma_r) = -2A$$

Multiply both sides by r

$$2\sigma_r r + r^2 \frac{d}{dr}(\sigma_r) = -2A r$$

$$\frac{d}{dr}(r^2 \sigma_r) = -2A r$$

Integrating wrt r

CBS

$$r^2 \sigma_r = -Ar^2 + B$$

$$\sigma_r = -A + \frac{B}{r^2}$$

$$\text{From eqn 2. } \sigma_t = A + \frac{B}{r^2}$$



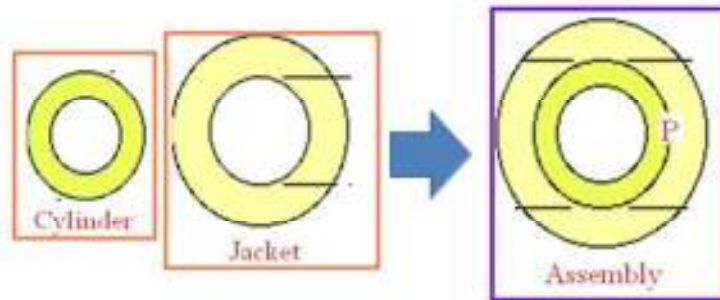
BCs

$$\sigma_r = P_i \quad \text{at} \quad r = \frac{d_i}{2}$$

$$\sigma_r = 0 \quad \text{at} \quad r = \frac{d_o}{2}$$

$$A = \frac{P_i d_i^2}{[d_o^2 - d_i^2]}$$

$$B = \frac{P_i d_i^2 d_o^2}{4[d_o^2 - d_i^2]}$$



δ_j = increase in inner diameter of jacket
 δ_c = decrease in outer diameter of cylinder

Tangential Strain in Outer cylinder (Jacket)

$$(\epsilon_t)_j = \frac{\text{change in circumference}}{\text{original circumference}} = \frac{\pi(D_2 + \delta_j) - \pi D_2}{\pi D_2} = \frac{\delta_j}{D_2}$$

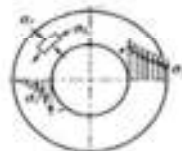
Tangential Strain in Inner cylinder (Cylinder)

$$(\epsilon_t)_c = \frac{\text{change in circumference}}{\text{original circumference}} = \frac{\pi(D_2 - \delta_c) - \pi D_2}{\pi D_2} = -\frac{\delta_c}{D_2}$$

$$(\epsilon_t)_c = \frac{1}{E} [\sigma_t - \mu \sigma_r]$$

$$\delta_c = \frac{D_2}{E} [\sigma_t - \mu \sigma_r]$$

$$\sigma_t = + \frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} \quad \sigma_r = -P$$



$$\delta_c = \frac{D_2 P}{E} \left[\frac{(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)} + \mu \right]$$

Compressive Strain in Inner cylinder

$$(\epsilon_t)_c = \frac{\pi D_2 - \pi(D_2 - \delta_c)}{\pi D_2} = \frac{\delta_c}{D_2}$$

$$(\epsilon_t)_c = \frac{1}{E} [\sigma_t - \mu \sigma_r]$$

$$\delta_c = \frac{D_2}{E} [\sigma_t - \mu \sigma_r]$$

$$\sigma_t = - \frac{P(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)} \quad \sigma_r = -P$$



$$\delta_c = - \frac{D_2 P}{E} \left[\frac{(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)} - \mu \right]$$

20 t

$$t = \frac{P_i D_o}{2\sigma_L}$$

Q1) The piston rod of a hydraulic cylinder exerts an operating force of 10 kN. The friction due to piston packing & stuffing box is equivalent to 10% of operating force. The pressure in the cylinder is 10 MPa. The cylinder is made of cast iron FG200 & the factor of safety is 5. Determine the diameter & thickness of the cylinder.

Ans). The cylinder is brittle \therefore we will use Lame's equation.

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma_t + P_i}{\sigma_L - P_i}} - 1 \right] \quad \text{--- (1)}$$

The total force on the piston,

$$P = 10 \times 10^3 + \underbrace{\frac{10}{100} (10 \times 10^3)}_{\text{friction}} = 11,000 \text{ N}$$

Let D_i be the internal dia of the cylinder.
 $\therefore P_i = \frac{P}{A}$ [$P_i = 10 \text{ MPa}$ (Given).]

$$10 = \frac{11000}{\frac{\pi}{4} D_i^2} \quad \therefore D_i = 37.42 \text{ mm}$$

$$D_i \approx 40 \text{ mm} \quad \text{--- Ans.}$$

Put all the values in eqⁿ 1.

$$t = \left(\frac{40}{2}\right) \left[\sqrt{\frac{\sigma_t + 10}{\sigma_t - 10}} - 1 \right]$$

$$\sigma_t = \frac{200}{\text{fos}} = \frac{200}{5} = 40 \quad \therefore \text{put in above.}$$

$$t = \left(\frac{40}{2}\right) \left[\sqrt{\frac{40 + 10}{40 - 10}} - 1 \right]$$

$$t = 5.82 \text{ mm} \approx 6 \text{ mm} \quad \text{--- Ans}$$

e) The inner diameter of a cylindrical tank for liquified gas is 250 mm. The gas pressure is limited to 15 MPa. The tank is made of plain CS 10C4 ($\sigma_{ut} = 340 \text{ N/mm}^2$ & $\mu = 0.27$) and the factor of safety is 5. Calculate the cylinder wall thickness.

$$\text{Ans) } \sigma = \frac{\sigma_{ut}}{\text{fos}} = \frac{340}{5} = 68 \text{ N/mm}^2$$

Tank is made of Ductile material, so using Clavarino's equation.

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma + (1-2\mu) P_i}{\sigma - (1+\mu) P_i}} - 1 \right]$$

$$= \frac{250}{2} \left[\sqrt{\frac{68 + [1 - 2(0.27)](15)}{68 - [1 + 0.27](15)}} - 1 \right]$$

$$t = 29.62$$

$$t = 30 \text{ mm}$$

A seamless steel pipe of 100 mm internal diameter is subjected to internal pressure of 12 MPa. It is made of steel ($S_{yt} = 230 \text{ N/mm}^2$ & $\mu = 0.27$) & the factor of safety is 2.5. Determine the thickness of the pipe.

Ans) The pipe has open ends so Brinley eqn is applicable

$$t = \frac{D_i}{2} \left[\sqrt{\frac{\sigma + (1-\mu)P_i}{\sigma - (1+\mu)P_i}} - 1 \right]$$

$$\sigma = \frac{S_{yt}}{f.o.s} = \frac{230}{2.5} = 92 \text{ N/mm}^2.$$

$$\therefore t = \frac{100}{2} \left[\sqrt{\frac{92 + (1-0.27)(12)}{92 - (1+0.27)(12)}} - 1 \right]$$

$$t = 7.29 \approx 8 \text{ mm.}$$

Cylinders with external pressure.

we had derived,

$$\sigma_r = C_1 + \frac{C_2}{r^2}$$

$$\sigma_\theta = -C_1 + \frac{C_2}{r^2}$$

put the boundary condns

$$\sigma_\theta = P_0 \quad \text{when } r = \frac{D_o}{2}$$

$$\sigma_r = 0 \quad \text{when } r = \frac{D_i}{2}$$

which yields,

$$C_1 = \frac{-P_0 D_o^2}{(D_o^2 - D_i^2)}$$

$$C_2 = - \frac{P_0 D_i^2 D_o^2}{4 (D_o^2 - D_i^2)}$$

$$\therefore \sigma_r = - \frac{P_o D_o^2}{(D_o^2 - D_i^2)} \left[1 - \frac{D_i^2}{4r^2} \right]$$

$$\sigma_t = - \frac{P_o D_o^2}{(D_o^2 - D_i^2)} \left[1 + \frac{D_i^2}{4r^2} \right]$$

At inner surface of cylinder.

$$r = \frac{D_i}{2}$$

$$\therefore \sigma_r = 0$$

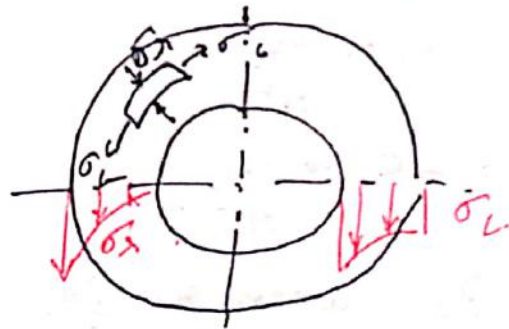
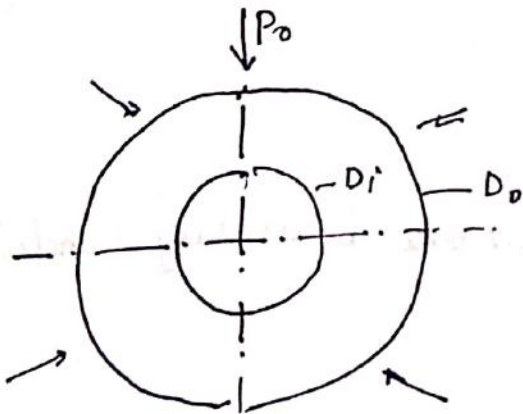
$$\sigma_t = - \frac{2 P_o D_o^2}{(D_o^2 - D_i^2)}$$

At the outer surface

$$r = \frac{D_o}{2}$$

$$\sigma_r = - P_o$$

$$\sigma_t = - \frac{P_o (D_o^2 + D_i^2)}{(D_o^2 - D_i^2)}$$



Autofrettage

Autofrettage is a process of pre-stressing the cylinder before using it in service.

Its used in case of high-pressure cylinders and gun barrels.

When the cylinder is subjected to internal pressure, circumferential stress at the (σ_c) inner surface limits the pressure capacity of the cylinder.

In prestressing process, residual compressive stresses are developed at the inner surface. When cylinder is loaded in service, the residual compressive stresses at the inner surface begin to decrease, become zero & finally become tensile, as the pressure is gradually increased.

There are three methods of pre-stressing the cylinders.

- 1) A compound cylinder, consists of two concentric cylinders with outer cylinder shrunk on inner one. This induces compressive stresses in the inner cylinder.
- 2) Overload the cylinder before its put in service. The overloading pressure is adjusted in such a way that a portion of cylinder near the inner dia. is subjected to stresses in plastic range, while the outer portion is still in elastic range. When pressure is released, the outer portion ^{which was} ~~is still~~ in the elastic range, starts contracting exerting pressure on inner portion.

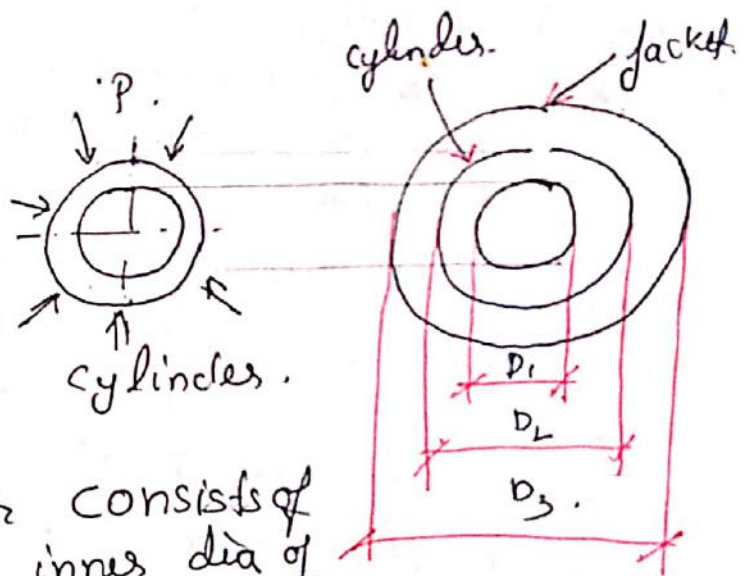
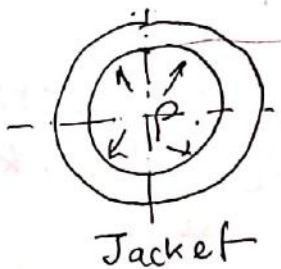
This induces residual compressive stresses at the inner surface.

iii) A wire under tension is closely wound around the cylinder, which results in residual compressive stresses.

Advantages of Autofrettage.

- i) It increases the pressure capacity of the cylinder.
- ii) The residual compressive stresses close the cracks within the cylinder resulting in increased endurance strength.

Compound cylinder



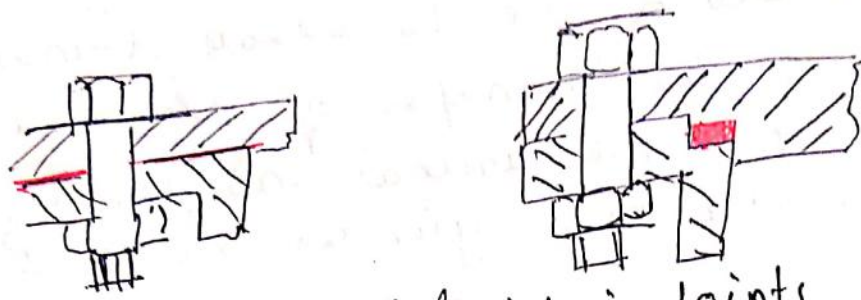
A compound cylinder consists of cylinder & jacket. The inner dia of the jacket is slightly smaller than the outer dia of the cylinder, when the jacket is heated, it expands sufficiently to move over the cylinder. As the jacket cools, it tends to contract on to the inner cylinder, which induces residual compressive stresses. There is a shrinkage pressure, P , between the cylinder & the jacket. The pressure P tends to contract the cylinder & expand the jacket.

A gasket is a device used to create & maintain a barrier against the transfer of fluid across the mating surfaces of a mechanical assembly. Its use is in static joints, such as cylinder block, cylinder head.

There are two types of gaskets, metallic & non-metallic. Metallic gaskets consist of sheet of lead, copper or aluminium. Non-metallic gaskets are made of asbestos, cork, rubber or plastics. Metallic gaskets are used for high temperature & high pressure.

Metallic gasket takes a permanent set when compressed in assembly. & there is no recovery to compensate for separation of contact forces. They are also susceptible to corrosion & chemical atmosphere, their performance depends on surface finish of contacting surfaces.

Asbestos gaskets have excellent resistance to crushing loads. & cutting action, they also possess dimensional stability, they are used in cylinder head, valves & steam pipe fittings. Vulcanized compounds of rubber & cork are employed as gaskets in steam lines, combustion chambers & chemical environment but they are affected by fungus & ~~impure~~ alkalis, they can flow in imperfections.



fig! use of gaskets in joints

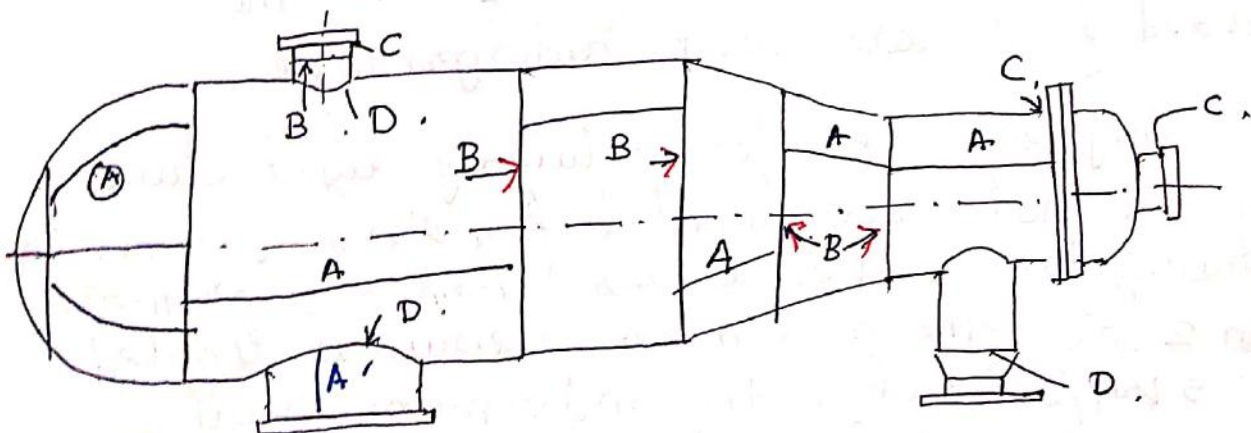
Unfired Pressure vessels,

An unfired pressure vessel is defined as a vessel or a pipeline for carrying, storing or receiving steam, gases or liquids, at pressures above the atmospheric pressure.

The Indian standard code for pressure vessels gives the design procedure for welded pressure vessels that are made of ferrous materials and subjected to internal pressure from 1 kgf/cm^2 to 200 kgf/cm^2 .

Small pressure vessels with diameters less than 150 mm or water containers with capacities of less than 500 litres do not come under the scope of this code. The code does not include steam boilers, nuclear pressure vessels or hot water storage tanks.

There are four categories of welded joints - A, B, C & D. The term category defines only the location of welded joint



Category A: Longitudinal welded joints within the main shell, communicating chambers & nozzle circumferential joints connecting the end closure to the main shell, any welded joint in spherical or formed head.

Category B: Circumferential welded joints in the main shell, ~~circumferential~~ communicating chambers or nozzles.

Category C: welded joints connecting flanges and flat heads to the main shell.

Category D: welded joints connecting communicating chambers & nozzles to the main shell.

Pressure vessels are classified into three groups - Class 1, Class 2 & Class 3.

Class 1: This group of pressure vessel ~~is~~ are used to contain lethal & toxic substances eg: hydrocyanic acid, carbonyl chloride. They are also used when the operating temperature is less than -20°C , they are fully radiographed (weld).

Class 2: They are same as class 1 but the welded joints are spot radiographed.

Class 3: They are used for relatively light duties. They are not recommended for service when the operating temperature is less than 0°C or more than 250°C . The maximum pressure is limited to 17.5 kgf/cm^2 while the maximum shell thickness is limited to 16 mm . They are usually made from carbon & low alloy steels, they are not radiographed.

Pressure

There are three terms related to pressure, working pressure, design pressure & hydrostatic pressure.

Working pressure

The maximum working pressure is that which is permitted for the vessel in operation. It is the pressure required for the processes that are carried out inside the pressure vessel.

Design pressure

The pressure used in design calculations for quantities as shell thickness & also in the design of other attachments, like nozzles and openings.

$$\text{Design pressure} = 1.05 (\text{maximum working pressure}),$$

Hydrostatic Test pressure

The pressure vessel is finally tested by hydrostatic test.

$$\text{Hydrostatic test pressure} = 1.3 (\text{design pressure}).$$

Weld joint Efficiency.

Pressure vessels are fabricated from steel plates welded together by the fusion welding process. The term weld joint efficiency is often used in pressure vessel design. It is defined as the ratio of the strength of the welded joint to the strength of the plates.



Double welded butt joint



Single welded butt joint with backing strip



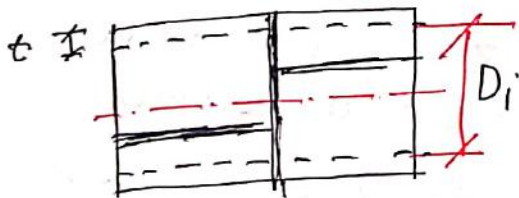
Single welded butt joint without backing strip

Table: weld joint efficiency.

Type of welded joint	weld joint efficiency (η)		
	fully radiographed.	Spot Radiographed.	Not examined.
a) Double welded butt joint with full penetration	1	0.85	0.70
b) Single welded butt joint with backing strip	0.9	0.8	0.65
c) Single welded butt joint without backing strip.	—	—	0.60

Thickness of cylindrical & spherical shells

The thickness of cylindrical or shell subjected to internal pressure is given by.



$$t = \frac{P_i D_i}{2 \sigma_t \eta - P_i} + CA,$$

for spherical shell.

$$t = \frac{P_i D_i}{4 \sigma_t \eta - P_i} + CA,$$

t = minimum thickness of the shell plate (mm).

P_i = design pressure (MPa).

D_i = inner dia of the shell (mm).

σ_t = allowable stress for the plate material

η = weld joint efficiency.

CA = corrosion allowance (mm).

$$\sigma_t = \frac{\text{Yield strength (or 0.2\% proof stress)}}{1.5}$$

$$\sigma_t = \frac{\text{Ultimate tensile strength}}{3.0}$$

fos of 1.5 or 3 in the above expression is used under the following two conditions:

- i) The pressure vessel is operating at room temperature.
- ii) The pressure inside the vessel is not fluctuating

The walls of the pressure vessel are subjected to thinning due to corrosion, which may be of the following forms.

- i) Chemical attack, where the metal is dissolved by a chemical reagent
- ii) Rusting due to ~~air~~ & moisture.
- iii) Erosion, where a reagent flows over the wall surface at high velocities
- iv) Scaling or high temperature oxidation.

A minimum CA of 1.5 mm is recommended unless a protective lining is employed.

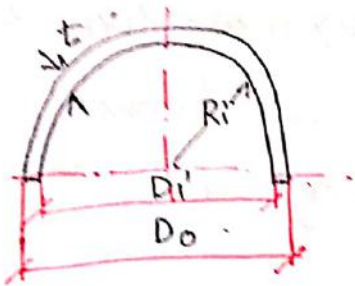
End closures:

formed heads are used as end closures for cylindrical pressure vessels. There are two types of end closures.

- domed heads.
- conical heads.

The domed heads are further classified as.

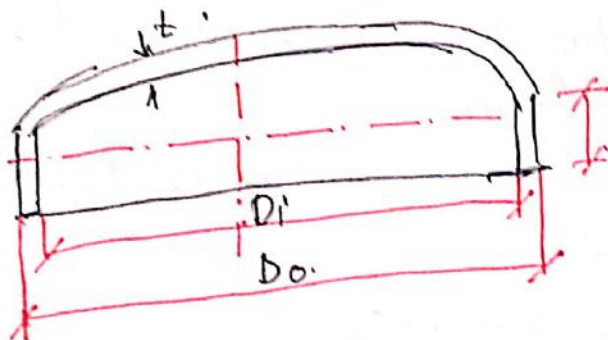
- a) Hemispherical head (min. thickness & min weight).



$$t = \frac{P_i R_i}{2\sigma_t \eta - 0.2 P_i} + CA.$$

R_i = Inner radius of the cylindrical shell.

- b) Semi-ellipsoidal head.



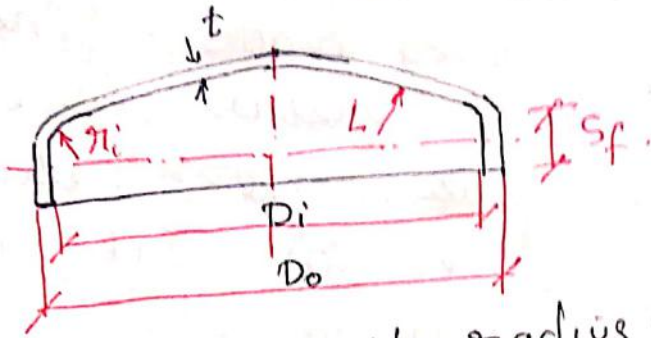
ratio of major axis to the minor axis is taken as 2:1.

$$t_r = \frac{P_i D_i}{2\sigma_t \eta - 0.2 P_i} + CA.$$

The thickness of the semi-ellipsoidal head is almost twice of the corresponding hemispherical head; the material cost is also more, however, due to shallow dished shape the forming cost is reduced.

$$S_f = 3t \text{ or } 20 \text{ mm (whichever is more)}$$

c) Torispherical head.



r_i → Knuckle radius.
 L → Crown radius.

Torispherical are extensively used as end closures for a large variety of cylindrical pressure vessels. They require less forming than semi-ellipsoidal heads. Their main drawback is the local stresses at the two discontinuities, namely the junction between the crown & the knuckle radius & the junction between the knuckle radius & the cylindrical shell. The thickness is given

$$t = \frac{0.885 P_i L}{\sigma_t \eta - 0.1 P_i} + CA.$$

$$r_i = 0.06 L.$$

Crown radius L should not be greater than outside dia of the cylindrical shell.

$$L < D_o.$$

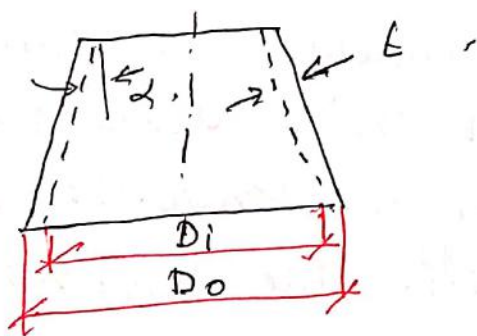
Hemispherical & semi-ellipsoidal heads are used for all tall vertical towers because they are practically free from discontinuities.

Space is not a limiting factor for vertical pressure vessels.

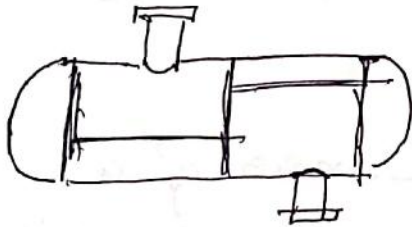
Torispherical heads are more economic than other types of domed heads. They are used for horizontal pressure vessels such as tankers for water, milk, petrol, diesel or kerosene. They are also used for small vertical pressure vessels.

The thickness of conical head or section is given by

$$t = \frac{P_i D_i}{2 \cos \alpha (\sigma_t \eta - 0.6 P_i)} + CA$$



The cylindrical shell, shown in fig is subjected to an operating pressure of 0.75 MPa. The yield strength of the plate material is 200 N/mm^2 & the corrosion allowance 3 mm , spot radiographed double welded butt joints are used to fabricate the shell, whose internal diameter is 2.5 m . Torispherical heads, each with a crown radius of 2 m , are used as end closure. Determine the thickness of the cylindrical shell and the torispherical head.



A). Design pressure P_i .

$$P_i = 1.05(0.75) = 0.7875 \text{ MPa}$$

$$\eta = 0.85$$

$$\sigma_t = \frac{S_{yt}}{f_{os}} = \frac{200}{1.5} = 133.33 \text{ N/mm}^2$$

$$\text{we have } t = \frac{P_i D_i}{2\sigma_t \eta - P_i} + CA.$$

$$t = \frac{(0.7875 \times 2500)}{(2 \times 133.33 \times 0.85) - 0.7875} + 3.$$

$$t = 11.72 \approx 12 \text{ mm}.$$

The thickness of torispherical head.

$$t = \frac{0.885 P_i L}{\sigma_t \eta - 0.1 P_i} + C A.$$

$$t = \frac{0.885 \times 0.7875 \times 2000}{[133.33 \times 0.85] - 0.1 \times 0.7875} + 3.$$

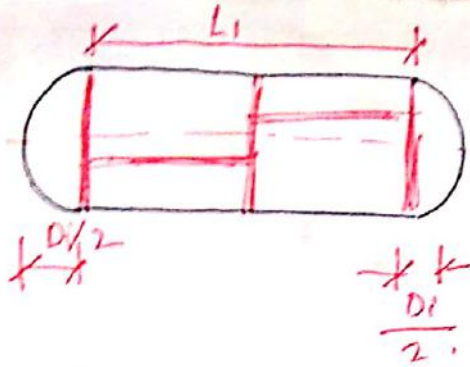
$$t = 15.3$$

i.e. $t \approx 16 \text{ mm}$.

$$\begin{aligned} \text{Knuckle radius} &= 0.06 L \\ &= 0.06 \times 2000 \\ &= 120 \text{ mm}. \end{aligned}$$

a) A horizontal pressure vessel consist of a cylindrical shell enclosed by hemispherical ends. The volume capacity of the vessel should be approximately 2 m^3 & the length should not exceed 3 m . Assuming the thickness negligibly small compared with overall dimensions of the vessel, determine the internal dia & the length of the cylindrical shell.

The pressure vessel is fabricated from steel plates with yield strength of 225 N/mm^2 . The weld joint efficiency factor is 0.85 & corrosion allowance 2 mm . The pressure vessel is subjected to an operating pressure of 2 MPa . Calculate the thickness of the cylindrical shell & the hemispherical end closures.



Its given that
 $V = 2 \text{ m}^3$.

$$i.e. V = \underbrace{\frac{\pi}{4} D_i^2 L_1}_{\text{cylindrical portion}} + \underbrace{\frac{\pi}{6} (D_i)^3}_{\text{spherical}} \quad \text{--- (1)}$$

Let's apply the geometrical constraints

$$L = D_i + L_1$$

But $L = 3 \text{ m}$ (max) as given in question.

$$\therefore L_1 = 3 \text{ m} - D_i \quad \text{--- (2)}$$

Put (2) in (1).

$$2 = \frac{\pi}{4} D_i^2 (3 - D_i) + \frac{\pi}{6} D_i^3$$

$$2 = \frac{\pi}{4} (3D_i^2 - D_i^3) + \frac{\pi}{6} D_i^3$$

$$2 = \pi \left[\frac{3}{4} D_i^2 - \frac{D_i^3}{4} + \frac{D_i^3}{6} \right]$$

$$\frac{2}{\pi} = \frac{3}{4} D_i^2 - 0.083 D_i^3$$

$$+0.083 D_i^3 - \frac{3}{4} D_i^2 + \frac{2}{\pi} = 0$$

$$D_i^3 - 9 D_i^2 + 7.67 = 0$$

$$D_i = 0.97 \text{ m}$$

$$\begin{array}{c} + \quad + \quad + \\ \hline 0 \quad 0.5 \quad 0.75 \quad 1 \end{array}$$

$$\begin{array}{c} + \quad + \quad \dots \\ \hline 0.9 \quad 0.95 \quad 1 \end{array}$$

$$\therefore L_1 = 3 - 0.97 \\ = 2.03 \text{ m,}$$

$$\therefore V = \frac{\pi}{4} (0.97)^2 \times 2.03 + \frac{\pi}{6} \times (0.97)^3 \\ = 1.5 + 0.492,$$

$$\boxed{V = 1.992 \text{ m}^3}$$

we have thickness of cylindrical shell.

$$t = \frac{P_i D_i}{2\sigma_t \eta - P_i} + CA,$$

$$P_i = \text{Design pressure} = 1.05(2) = 2.1 \text{ N/mm}^2$$

$$D_i = 970 \text{ mm}$$

$$\sigma_t = \frac{255}{1.5} = 170 \text{ N/mm}^2$$

$$\eta = 0.85$$

$$CA = 2.$$

$$\therefore t = \frac{(2.1 \times 970)}{(2 \times 170 \times 0.85)} + 2$$

$$\boxed{t = 9.04 \approx 10 \text{ mm.}}$$

Thickness of hemispherical end closures

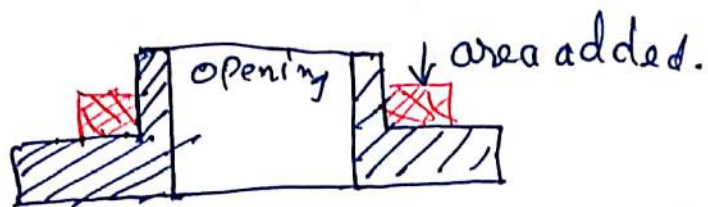
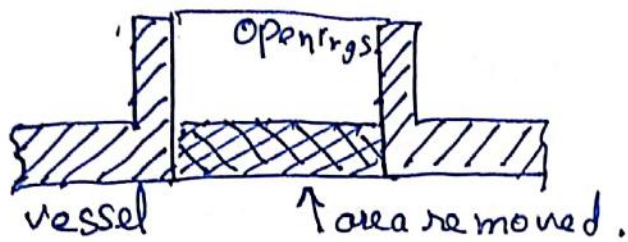
$$t = \frac{P_i R_i}{2\sigma_t \eta - 0.2P_i} + CA$$

$$= \frac{2.1 \times 485}{(2 \times 170 \times 0.85) - (0.2 \times 2.1)} + 2 = 5.52$$

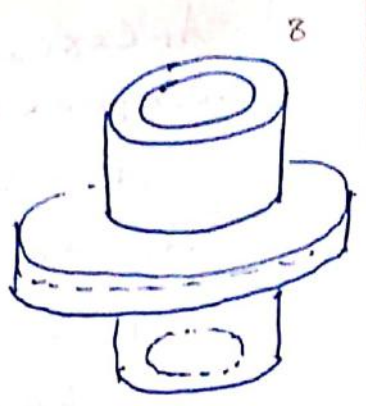
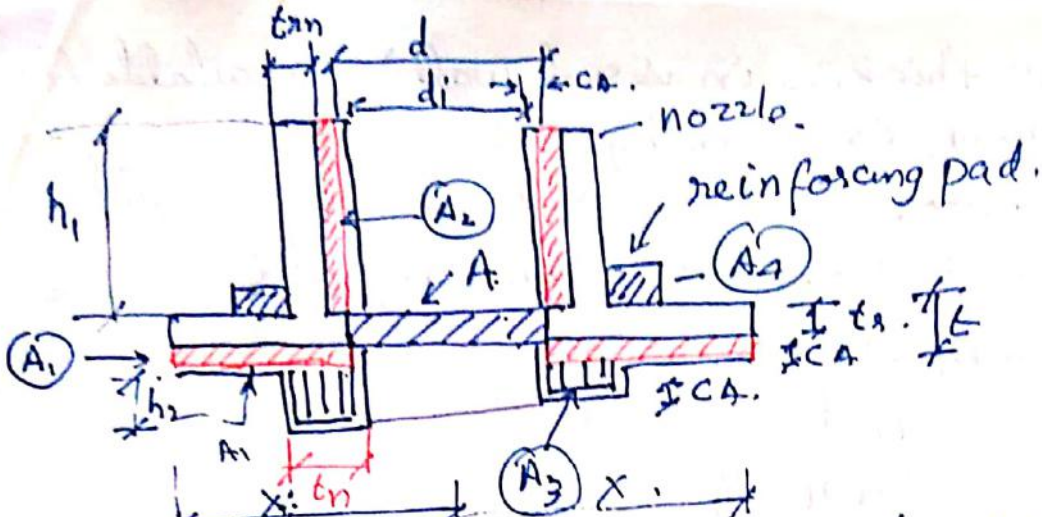
$$\therefore \boxed{t = 6 \text{ mm}}$$

Openings in pressure vessel.

Openings are provided in the pressure vessel for pipe connection, man hole, hand hole, pressure gauges, temperature gauges & safety valve. They are mostly designed by area compensation method.



The area is added in the form of circular plate (reinforced) around the opening. In this method we are considering cross-sectional area in the form of rectangular strip. It's not compensation of volume of metal.



It is not always necessary to replace the actual removed area of the metal. The plate of shell & nozzle are usually thicker than that required to withstand pressure

$$A = d t_s \quad \text{--- (1)}$$

A = area of metal removed in corroded condition.

d = inner dia of opening in corroded condition,
 $= d_i + 2CA$

d_i = inner dia of nozzle.

t_s = thickness required of cylindrical shell.

$$t_s = \frac{P_i D_i}{2\sigma_c \eta - P_i}$$

$$x = d \quad \text{or} \quad x = \left[\frac{d_i}{2} + t + t_n - 3CA \right] \quad \text{(whichever is maximum)}$$

$$h_1 \text{ or } h_2 = 2.5 (t - CA)$$

$$h_1 \text{ or } h_2 = 2.5 (t_n - CA) \quad \text{(whichever is minimum)}$$

t = total thickness of wall of cylindrical shell.
 (cm or in)

t_n = total thickness of nozzle wall.

A_1 (excess thickness in vessel wall), available for reinforcement, is given by,

$$A_1 = (2x+d)(t - t_n - CA) \quad - (2)$$

A_2 excess thickness area in nozzle

$$A_2 = 2h_1(t_n - t_{rn} - CA) \quad - (3)$$

$$t_{rn} = \frac{P_i d_i}{2\sigma_t \eta - P_i}$$

A_3 the area of inside extension of nozzle.

$$A_3 = 2h_2(t_n - 2CA) \quad - (4)$$

Total area for reinforcement = $A_1 + A_2 + A_3$.

when $A_1 + A_2 + A_3 \geq A$.

no pad is required (enforcing pad).

But if not then,

$$A_4 = A - (A_1 + A_2 + A_3) \quad - (5)$$

Some times A is used for opening to avoid detail calculations.

This results in oversized reinforcement

A pressure vessel consists of a cylindrical shell with an inner dia of 1500 mm and a thickness of 20 mm. It is provided with a nozzle of inner diameter of 250 mm and thickness 15 mm. The yield strength of the material for the shell & nozzle is 200 N/mm^2 & the design pressure is 2.5 MPa. The extension of the nozzle inside the vessel is 15 mm. The corrosion allowance is 2 mm, while the weld joint efficiency is 0.85. Neglecting the area of welds, determine whether or not a reinforcing pad is required for the opening. If so determine the dimensions of pad made from a plate of 15 mm thickness.

$$\text{Ans) } \sigma_t = \frac{S_{yt}}{f_{os}} = \frac{200}{1.5} = 133.33 \text{ N/mm}^2$$

$$t_r = \frac{P_i D_i}{2 \sigma_t \eta - P_i}$$

$$t_r = \frac{2.5 \times 1500}{2(133.33)(0.85) - 2.5} = 16.73 \text{ mm}$$

$$d = d_i + 2CA = 250 + 2(2) = 254$$

$$A = d t_r = 254 \times 16.73 = 4249.42 \text{ mm}^2$$

$$t_{rn} = \frac{P_i d_i}{2 \sigma \eta - P_i} = \frac{2.5 \times 250}{(2 \times 133.33 \times 0.85) - 2.5}$$

$$t_{rn} = 2.79 \text{ mm}$$

$$x = d = 254 \text{ mm}$$

$$X = \left[\frac{d_i}{2} + t + t_n - 3CA \right]$$

$$= [125 + 20 + 15 - 6] = 154 \text{ mm.}$$

$$\therefore X = 254 \text{ mm.}$$

$$h_1 = 2.5 (t - CA) = 2.5 (20 - 2) = 45 \text{ mm}$$

$$h_1 = 2.5 (t_n - CA) = 2.5 (15 - 2) = 32.5$$

$$\therefore h_1 = 32.5 \quad h_2 = 15 \text{ mm}$$

$$A_1 = (2x - d) (t - t_n - CA)$$

$$= (2 \times 254 - 254) (20 - 16.73 - 2)$$

$$A_1 = 322.58 \text{ mm}^2$$

$$A_2 = 2h_2 (t_n - t_{nn} - CA)$$

$$A_2 = 2 \times 32.5 \times (15 - 2.79 - 2)$$

$$A_2 = 663.65 \text{ mm}^2$$

$$A_3 = 2h_2 (t_n - 2CA)$$

$$= 2 \times 15 (15 - 4)$$

$$A_3 = 330 \text{ mm}^2$$

$$A_1 + A_2 + A_3 = 322.58 + 663.65 + 330$$

$$= 1316.23 \text{ mm}^2$$

$$\therefore A > (A_1 + A_2 + A_3)$$

\therefore Pad is necessary (Reinforcement).

$$\therefore A - (A_1 + A_2 + A_3) = A_4$$

$$\therefore A_4 = 4249.42 - 1316.23 = 2933.19 \text{ mm}^2$$

Pad thickness = 15 mm (given).

$$\therefore w = \frac{2933.19}{15} = 195.55$$

ASSIGNMENT- DESIGN OF CYLINDERS AND PRESSURE VESSEL

- 1-What is autofrettage? Explain any one method of pre stressing the cylinders
- 2- Derive Birnie's equation. Explain under what conditions it is used.
- 3- Explain the basic principle of the area compensation method. Also explain area compensation for nozzle with its equations
- 4-The piston rod of a hydraulic cylinder exerts an operating force of 12kN. The friction due to piston packing and stuffing box is equivalent to 10% of the operating force. The pressure in the cylinder is 10 MN/m². The cylinder is made of cast iron FG 200 and factor of safety is 5. Determine the diameter and thickness of cylinder.
- 5- A hydraulic cylinder with closed ends is subjected to an internal pressure of 15 MPa. The inner and outer diameters of the cylinder are 200 mm and 240 mm respectively. The cylinder material is cast iron FG 300. Determine the factor of safety used in design. If the cylinder pressure is further increased by 50%, what will be the factor of safety?
- 6-A pressure vessel consists of cylinder shell with 2m inside diameter and 10 mm thickness. It is subjected to design pressure 0.75 MN/m² and having nozzle of inner diameter 300 mm and wall thickness of 10mm. The corrosion allowance is 2 mm and weld efficiency is 0.85. The extension of nozzle inside and outside the shell is 15mm. Take $S_{yt} = 210$ MPa. A reinforcing pad of 10mm thick plate is provided for opening. Factor of safety = 1.5. Determine the dimensions of reinforcing pad.
- 7- What are the objectives of providing openings in pressure vessel
- 8- What are types of end closure for cylindrical vessel? State the design procedure of hemispherical head.
- 9- Explain the various categories of the welded joints used in unfired pressure vessel
- 10- Derive the expressions to find principal stresses at the inner surface of a thick cylinder.